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Invest as you go: How public health investment keeps pension systems healthy*

Paolo Melindi-Ghidi[†] Willem Sas[‡]

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Abstract

Better health not only boosts longevity in itself, it also postpones the initial onset of disability and chronic infirmity to a later age. In this paper we examine the potential effects of such 'compression of morbidity' on pensions, and introduce a health-dependent dimension to the standard pay-as-you-go (PAYG) pension scheme. Studying the long-term implications of such a system in a simple overlapping generations framework, we find that an increase in public health investment can augment capital accumulation in the long run. Because of this, the combination of health investment with a partially health-dependent PAYG scheme may in fact outperform a purely PAYG system in terms of lifetime welfare.

JEL Classification: I15, J26, O41.

Keywords: Health Investment, Disability Pension, Long-Term Care, PAYG Pension System, OLG Model.

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1 Introduction

Health care improvements have multiple effects on an increasingly ageing population, not in the least when it comes to disability incidence. Indeed, having lived a healthier life, we improve our chances on postponing the inevitable slide towards disability and eventual loss of autonomy. In this paper we study this evolution, and whether it can be harnessed to design more effective pension schemes.

Whereas the ongoing debate on pension systems tends to center on health improvements as a *cause* of unsustainable pension benefits, our take here will be entirely different. True enough, healthier people will live longer, which together with decreasing fertility rates mounts the pressure on the sort of 'pay-as-you-go' (PAYG) systems in place in most OECD countries.[1] Yet as we will show in our model, better health needn't always be a hurdle. By structurally rethinking the design of PAYG systems, health improvements can in fact take the heat off increasingly unsustainable pension liabilities, whilst adding to overall welfare at the same time.

The reason is simple, and due to what is known as *compression of morbidity* in medical terms. A healthier lifestyle nudges up the age at which initial disability or chronic infirmity sets in, outpacing any gains in longevity which also follow from improved health. This results in fewer years of disability across the board, as loss of autonomy is 'compressed' into an ever smaller time frame.[2] In other words, propped up public health investment dampens disability incidence more than it boosts longevity. If pensions were then to a larger extent conditional on health, by means of e.g. disability pensions or long-term care benefits, pensions could wind down even as longevity continues to rise. What is more, forward looking agents will align their saving decisions with this brand new pension arrangement, which could shore up capital accumulation and long-term economic growth.

To examine these dynamics, we set up a general equilibrium model where individual health and pension benefits are interlinked across time. We use a standard PAYG extension to the textbook overlapping generations (OLG) model introduced by Diamond (1965), and allow for two kinds of pension entitlements. A proportionally universal pension - similar to any PAYG scheme - and a conditional 'disability' pension which depends on individual health. The healthier the older generation in other words, the less pension benefits they will receive and vice versa. Crucially, whether the retired turn out healthier than their predecessors is endogenously determined by public health investment over earlier stages of life.

[1] The reason is that for each beneficiary pensioner there are fewer working contributors, a downward trend which is projected to accelerate (Pecchenino and Pollard, 2005; Cigno, 2007). See also United Nations (1998) or Cigno and Werding (2007) on increasing age-dependency ratios.

[2] Fries (1989) was first to coin the term, with many empirical follow-ups providing evidence. See e.g. Vita et al. (1998), Doblhammer and Kytir (2001), Hubert et al. (2002), or Fries et al. (2011). Faria (2015) concludes that compression of morbidity 'should be upgraded from a hypothesis to a theory', given the amount of evidence at hand.

Our main theoretical contributions are twofold. First, we find that an increase in public health investment can brace capital accumulation in the long run. Once the downwards effect of health improvements on pensions is brought into play, younger generations act on the stronger incentive to save so that capital levels rise alongside health investment. Up to a certain level of taxation, this second-round effect of health investment always offsets distortions caused by the tax hike financing the investment. Second, and because of this effect, combining health investment with a partially health-dependent PAYG pension scheme is shown to outperform a purely PAYG system. This in lifetime utility terms and at identical levels of tax burden.

Now, in a world where the importance of *long-term care* assistance[3] has grown together with the number of dependent elderly, our results offer some relief.[4] Although highly stylised, our model indeed captures the main elements of most long-term care arrangements currently in place. First, because of shifting family patterns and a failing private market, the brunt of elderly care has come to lie with the public sector.[5] Second, long-term care benefits are assigned on a *conditional* basis, usually by means of disability scales identifying various levels of dependency (e.g. the Katz scale). Third, most of the formal long-term care assistance of this kind is financed on a PAYG basis.

Moreover, since public expenditures on long-term care are projected to rise substantially in the future,[6] the lever of public health investment described in our model will bite all the more. Unlike a simple increase in pension contributions, beefing up public health investment in a budget neutral way could then keep pensions sustainable whilst improving overall welfare.

2 Background

This paper bridges two strands of literature on inter-generational concerns, both hinging on the stylised overlapping generations (OLG) framework pioneered by Diamond (1965).

Firstly, parsimonious OLG modeling has often been used to study the effectiveness of PAYG pensions. Indeed, changes in fertility rates and life expectancy over the last decades have fueled this debate, and are well suited to a simple overlapping generations setup. To this end, Diamond's model has been extended in various

[3] This kind of elderly care care can be administered both at home and in various kinds of institutions, including nursing homes and long-stay hospitals (Cremer et al., 2012).

[4] More than two out of five people aged 65 or older report having some sort of functional limitation which range from sensory, physical, mental, or self-care disabilities, to difficulties leaving home (see Pestieau and Ponthière (2010)).

[5] See e.g. Brown and Finkelstein (2007) on the insurance puzzle in the long term care private insurance market, or Pestieau and Sato (2008) on the case for public nursing PLUS.

[6] Cremer et al. (2012) predict a 115% rise in expenditures in the EU27 over the next 40 years. A lower bound estimate they emphasise, since future changes in the number of people receiving informal or no care are expected to deteriorate, yet assumed constant in their analysis.

directions to capture the (simultaneous) effects of increasing longevity and curtailed fertility on pension systems.[7]

Second, public health investment and its long-term implications have also been sized up from a stylised OLG perspective. Chakraborty (2004) for one, adapts the model of Diamond (1965) so that *longevity* endogenously depends on public health investment. Raising taxes to finance public health investments then improves survival probabilities of the elderly. Anticipating this longer lifespan, agents save more to uphold consumption at an older age, thereby boosting capital accumulation in the long run. Health investment thus turns out to stimulate growth and development. Fanti and Gori (2011a) arrive at the opposite outcome, by endogenising old-age *productivity* rather than longevity. Logically, agents will save less for old-age if they can still earn a decent living at that point.

In our model, neither old-age productivity nor life expectancy are endogenously linked to public health investment. Rather, it's the quality of life *during* old age which will depend positively on public health investment.[8] Given any stretch of old age, to what extent does reduced disability or chronic infirmity impact pension systems? To answer this question, and taking our cue from the various 'long-term care' arrangements in place in OECD countries, pensions in our model take health status during old age into account.[9] As such, and since health-dependent benefits of this kind are usually financed on a 'pay as you go' (PAYG) basis, our design can be seen as a 'long-term care' augmented version of the purely unfunded pension schemes modelled in the literature.

All in all, we are first to consider the relationship between partially health dependent pension systems and public investment in health. Shedding some light on the long-term implications of combining extended health care with conditional pensions, we cover several blind spots in the policy debate as well.

The paper proceeds as follows. Section 3 describes the characteristics of the model, and establishes equilibrium. Section 4 delves into the effect of a rise in health taxation on steady-state capital accumulation. Section 5 combines all of our findings to shed light on the potential welfare ramifications brought about by the kind of mixed pension system we propose. Section 6 concludes.

[7] See e.g. De La Croix and Michel (2002), Fanti and Gori (2010), Pestieau and Ponthière (2012), Fanti and Gori (2012), Fanti and Gori (2014), Cipriani (2014).

[8] Our focus then serves as a logical counterpart to other approaches where longevity was endogenously modelled, but health status during old-age kept constant. See e.g. Blackburn and Cipriani (2002), Chakraborty (2004), Bhattacharya and Qiao (2007), De La Croix and Ponthiere (2010), Jouvet et al. (2010), De La Croix et al. (2012), De La Croix and Licandro (2013), De La Croix and Licandro (2013), and Fanti and Gori (2014).

[9] See Norton (2000) or Cremer et al. (2012) for an overview of the (cross-country) variety in long-term care programs.

3 The Model

We consider a closed economy, populated by perfectly foresighted and identical individuals whose finite lifespan is divided up into two generations: youth (working period), and old age (retirement period). During each time period t the newly born generation of N_t individuals overlaps with the previous one, growing at an exogenous rate of $n \in (-1; +\infty)$, where $N_t = (1+n)N_{t-1}$. When young, agents have one unit of labor at their disposal which they supply to firms earning the competitive wage rate w_t. As soon as they retire, agents get by on accumulated savings as well as on pension benefits provided by the government.

Now, what sets our model apart is the introduction of a social security dimension consisting both of pension and health care elements, catering to a wider array of elderly needs and general medical risks. To finance this social security system, the government looks to the working generation. It levies a health tax τ_h on gross labour incomes, and takes out a social security contribution rate τ_p. Health tax revenues are marked out for public investments in the health of working generations,[10] whilst the social security contributions are used to finance the pensions and public services of the elderly.

3.1 Public health investment

As set out in our introduction, health status during old age is to a large extent related to the degree of public health investment in earlier periods of life. Introducing these dynamics to our model, old-age health status d_{t+1} at time $t+1$ will depend on the level of public investment in health h_t at time t. Following Blackburn and Cipriani (2002), we specify this relationship as follows:

$$d_{t+1} = \frac{d_0 + d_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}} \tag{1}$$

Like Chakraborty (2004), we focus on the simplified case where $\delta = 1$, $\Delta = 1$, $d_0 = 0$ and $0 < d_1 \leq 1$.[11] As a result, the health status function is given by the non-decreasing, concave function: $d_{t+1} = \frac{d_1 h_t}{1+h_t}$, satisfying the following properties: $d[0] = 0$, $lim_{h \to \infty} d[h] = d_1$ and $lim_{h \to 0} d^{'}[h] = d_1$. Assuming positive health investment, $h_t > 0$, old-age health status will fall between $d_{t+1} \in [0,1]$.

[10] Such investments can range from building hospitals, setting up new vaccination programmes or prevention campaigns, bankrolling scientific research projects, or quite simply extending existing medical services. See e.g. Chakraborty (2004) or Fanti and Gori (2014) for a similar approach.

[11] We set d_0, the minimum health level when old, equal to zero to allow for the realistic situation of complete non-self-sufficiency during old age. Exogenous medical progress (due to e.g. scientific research) is denoted by d_1, and as such captures the efficiency of public health investments on old-age health status. Parameters δ and Δ lastly, further define the effectiveness of public health investment. Notice that setting both $\Delta = \delta = 1$ implies a tractable monotonic and concave function. By contrast, Blackburn and Cipriani (2002), study an S-shaped function, with $\delta > 1$.

Public health investments h_t at time t are financed through an exogenous tax τ_h on the labour incomes of young workers at time t. For the sake of simplicity we assume a constant proportional tax on gross wages, so that $h_t = g[\tau_h w_t] = \tau_h w_t$.[12] The higher public health expenditures in other words, the higher health investments.

3.2 Health-dependent pensions and social security

The novelty of our model lies in the design of the pension system. The higher the loss of autonomy or degree of morbidity, the higher the old-age benefits, *and* vice-versa. We assume that total pension benefits at time t comprise a standard universal PAYG benefit p_t^u as well as a disability benefit p_t^d. While the former is independent from health status and universally attributed, the latter directly depends on health conditions d_t of the retired. The per pensioner benefit then reads as follows:

$$p_t = \rho p_t^u + (1-\rho) p_t^d[\delta(d_t, \tau_p)] \tag{2}$$

Where $0 < \rho < 1$ defines the share of social security contributions $\tau_p w_t(1+n)$ directed to universal pensions p_t^u, as opposed to revenues earmarked for other social security programs such as the disability pension p_t^d, set by $(1-\rho)$. We can then re-formulate (2) as:

$$p_t = \overbrace{\rho \tau_p w_t(1+n)}^{Universal\, pension} + \overbrace{(1-\rho)\tau_p w_t(1+n)\delta[d_t]}^{Disability\, pension} \tag{3}$$

Indeed, the standard PAYG system would be a particular case of our model where $\rho = 1$. Zooming in on the disability pension in (3) moreover, the relation $\delta[d_t]$ is vital. As a downwards function of health through d_t, the health-dependent feature of our pension scheme emerges here: when health improves expenditures on disability benefits decrease, starting from the initial level of $\tau_p w_t(1+n)$. We assume this function is inversely related to the health status of elderly at time t, such that: $\delta[d_t] = (1-d_t)$. Therefore, and through $d_t[h_{t-1}]$, per pensioner benefits in time period t are endogenously determined by public health investments in the previous period h_{t-1} as follows:

$$p_t = \overbrace{\rho \tau_p w_t(1+n)}^{Universal\, pension} + \overbrace{(1-\rho)\tau_p w_t \tau_p (1+n)(1-d_t[h_{t-1}])}^{Disability\, pension} \tag{4}$$

Lastly, when health improves and disability pensions begin to fall, the government will have increasingly more funds at its disposal to spend at will. These excess funds $(1-\rho)w_t\tau_p d_t(1+n)$ are defined as $g_t[d_t]$, and fully re-invested to compensate the elderly for incurred disposable income losses because of lower disability benefits. This could then range from spending on infrastructure (retirement homes, leisure

[12] Chakraborty (2004); Bhattacharya and Qiao (2007); Fanti and Gori (2011a, 2012) and Fanti and Gori (2014) use the exact same simplifying assumption.

centers geared towards the elderly,..), or non-cash benefits such as free access to public transport or university classes.[13] Summing up, the per pensioner budget constraint faced by the government in period t is then given by:

$$p_t[d_t] + g_t[d_t] = \tau_p w_t(1+n) \tag{5}$$

3.3 Individuals

The expected lifetime utility of perfectly foresighted individuals of generation t is expressed by a homothetic and separable utility function U_t, defined over consumption and public investment:[14]

$$U_t = ln[c_{1,t}] + \beta(ln[c_{2,t+1}] + v[g_{t+1}[d_{t+1}]]) \tag{6}$$

where $c_{1,t}$ denotes consumption at a young age, $c_{2,t+1}$ consumption when retired, and $g_{t+1}[d_{t+1}]$ utility received from public investment in the elderly. For reasons of simplicity, we assume sub-utility $v[.]$ to be linear so that $v[g_{t+1}[d_{t+1}]] = (1-\rho)w_{t+1}\tau_p d_{t+1}$.[15]

Young individuals join the workforce and offer their only unit of labour to firms, receiving a competitive wage w_t per unit of labour. This salary is taxed at time t to finance both health and social security system expenditures. Therefore, the budget constraint of the young agent at time t is given by:

$$c_{1,t} + s_t = w_t(1 - \tau_h - \tau_p); \tag{7}$$

Consequently, net income at a young age is used for consumption $c_{1,t}$ and saving s_t, with the overall tax rate at $(\tau_p + \tau_h) \in [0,1]$. Savings are deposited in a mutual fund accruing at a gross expected return of r^e_{t+1}. When old secondly, consumption is financed out of savings and expected social security. The budget constraint of an old agent born at time t then reads as:

$$c_{2,t+1} = s_t(1 + r^e_{t+1}) + p^e_{t+1} \tag{8}$$

With p^e_{t+1} the expected pension benefit as defined by (4). Substituting equations (4), (7) and (8) into (6) and maximizing U_t w.r.t. savings s_t, the optimal saving decision of an individual born in period t can easily shown to be:

$$s_t = \frac{\beta w_t(1 + r^e_{t+1})(1 - \tau_h - \tau_p) - (1+n)w^e_{t+1}[1 - d_{t+1}(1-\rho)]}{(1+\beta)(1 + r^e_{t+1})} \tag{9}$$

[13] Of course, such budgetary savings could also be used to lower pension contributions or government debt. For reasons of tractability, and because g_t doesn't affect our main findings in what follows, we omit this possibility here.

[14] We assign index 1 to the young households and index 2 to the old households.

[15] This is a non-restricting assumption. In fact, as long as sub-utility is positive, using convex or concave functional forms would not change our results.

Since we're interested in the long-term implications of public health investment, the role of the latter in partial equilibrium is illustrative. Deriving (9) with respect to health taxation τ_h yields:

$$\frac{\partial s_t}{\partial \tau_h} = \frac{(1+n)w^e_{t+1}\left[\frac{\partial d_{t+1}}{\partial \tau_h}\right] - \beta w_t(1+r^e_{t+1})}{(1+\beta)(1+r^e_{t+1})} \gtreqless 0 \tag{10}$$

What matters in (10) is the numerator, weighing up two effects on individual saving behaviour:

$$(1+n)w^e_{t+1}\left[\frac{\partial d_{t+1}}{\partial \tau_h}\right] \gtreqless \beta w_t(1+r^e_{t+1}) \tag{11}$$

On the right hand side of (11) we find the usual income effect which hollows out savings. Indeed, a higher health tax logically reduces the amount of disposable income available for consumption as well as savings. A second effect runs counter to the first however, as captured by the left hand side of (11). Here, health taxation nudges up health investment which leads to better health d_{t+1} at old-age. Since this in turn pulls down future claims on the entitlement system, perfectly foresighted individuals have an incentive to save and uphold old-age consumption. At play here is a substitution effect from young to old-age consumption.

Which of both effects wins out in general equilibrium will depend on the steady state wage and interest rate levels, and by consequence, on the capital stock. In the following sections we introduce production of goods and services to close the model, and derive precisely such general general equilibrium features.

3.4 Firms

Final goods are produced using a Cobb Douglas technology $Y_t = AK_t^\alpha N_t^{1-\alpha}$, with $\alpha \in (0,1)$. $A > 0$ represents exogenous technology productivity or total factor productivity. We define the production function in per capita terms $y = f(k_t) = Ak_t^\alpha$, with k_t defined as capital per unit of labor. Assuming capital fully depreciates at the end of each period and the price of output is normalised to unity, perfect competition in the goods market implies that both capital and labor are paid their respective marginal product, that is $w_t = (1-\alpha)Ak_t^\alpha$ and $r_t = \alpha Ak_t^{\alpha-1} - 1$. Given the initial capital stock k_0, competitive equilibria are characterized by a sequence of $\{k_t\}$ that satisfies equations $k_{t+1} = \frac{s_t N_t}{N_{t+1}}$.

3.5 Equilibrium

Combining the savings condition defined in (9) with (1), and after some algebraic manipulation, we obtain the following capital accumulation rule for $k_{t+1} = \frac{s_t N_t}{N_{t+1}}$:

$$k_{t+1} = \frac{\alpha k_t^\alpha (1 + \mathrm{c1} k_t^\alpha \tau_h)(1 - \tau_h - \tau_p)\beta \mathrm{c1}}{(1+n)(\mathrm{c2} - \mathrm{c1} k_t^\alpha \tau_h(\alpha(\mathrm{c3} - 1 - \beta) - \mathrm{c3}))} \tag{12}$$

With $\mathrm{c1} = (1-\alpha)A$, $\mathrm{c2} = \alpha(1+\beta)+\tau_p(1-\alpha)$ and $\mathrm{c3} = \tau_p(1-d_1(1-\rho))$. Steady states of the above dynamic path of capital accumulation are defined by $k_{t+1} = k_t = \bar{k}^*$. Since equation (12) is a first order non-linear equation, we are not able to derive an analytical formulation for the non-trivial steady states. We can however show that the zero equilibrium of the system is unstable, and prove the existence and stability of a non-trivial steady state $\bar{k}^* > 0$.

Proposition 1 *The dynamic system described by equation (12) possesses two steady states $\{0, \bar{k}^*\}$. The positive steady state $\bar{k}^* > 0$ is the only stable steady state.*

Proof See appendix 1.

4 Public health investment and capital accumulation

Having established equilibrium, we can now focus on our main point of interest: the long-term welfare implications of combining a health-dependent pension scheme with health investment. In this light, deriving the effect of a rise in health investments on the steady-state level of capital is a necessary first step. Capital accumulation influences wages, interest rates, and thus inevitably defines long-term outcomes.

Indeed, such a comparative statics exercise is far from trivial as pointed out above, and expressed by (9). Higher health investments imply higher health taxes, which take an immediate bite out of disposable income, in turn discouraging savings and eroding the capital stock. Yet the partial equilibrium effect also works in the opposite direction, as health conditions during old-age improve because of health investment, which encourages saving. What we find is that when health taxation remains below a certain threshold level and the capital stock is high, the latter effect wins out in general equilibrium.

Proposition 2 *If $\bar{k}^* > \tilde{k}$, with $\tilde{k} = \left(\frac{\tau_p + \alpha(1-\tau_p+\beta)}{A(\alpha-1)^2 d_1(1-\tau_p)\tau_p(1-\rho)}\right)^{\frac{1}{\alpha}}$, then there exists a positive threshold $\bar{\tau}_h \in (0,1)$ such that an increase in τ_h has an ambiguous effect on the steady state level of capital $\bar{k}^*$: positive when $0 < \tau_h < \bar{\tau}_h$, negative otherwise. If $\bar{k}^* \leq \tilde{k}$ then an increase in τ_h always has a negative effect on the steady state level of capital $\bar{k}^*$.*

Proof See Appendix 2.

When $\tau_h < \bar{\tau}_h$ and $\bar{k}^* > \tilde{k}$, the downwards pressure of health improvements on pensions induces younger generations to save more, so that capital accumulation rises. The resulting higher wages translate into even more health investment -ceteris paribus with regard to the value of the health tax τ_h- which in turn improves health conditions of the elderly. This sparks off an indirect general equilibrium feedback effect which encourages saving even more, and serves as a catalyst to accumulate capital down the line. As a result, steady-state output per worker increases.[16]

However, this multiplier effect is only triggered under certain conditions. If the government sets a tax rate $\tau_h > \bar{\tau}_h$ which is too distortive, investment in public health impedes capital accumulation in the long run. A lower capital stock $\bar{k}^* < \tilde{k}$ also plays its part. To understand these conditions, we adjust expression (11) for steady-state values and simplify:

$$(1+n)\left(\frac{\partial d[h]}{\partial \tau_h}\right) \gtrless \beta(1+r) \tag{13}$$

Now, since $d[h]$ is concave in τ_h, higher values of τ_h will lessen the chances for the substitution effect on the left of (13) to outweigh the income effect on the right. As the sign flips in the opposite direction when τ_h jumps over $\bar{\tau}_h$, individuals start saving *less* after a health tax hike. Indeed, health investment in this case only leads to minor health gains, and very small reductions in future pensions. These are readily offset by the disposable income cuts, which remain the same on the margin. Similarly, lower steady-state capital levels will also tilt expression (13) in favour of the right hand side, since smaller capital stocks generate higher interest rates and lower wages.[17]

To illustrate how the steady state level of capital responds to an increase in the health tax rate, we perform a very simple numerical analysis in Table 1. When $\tau_h = 0$, we get a steady state level of capital $\bar{k}^* = 2.4316$, a threshold $\tilde{k} = 0.4058$ and a threshold $\bar{\tau}_h = 0.021$. As the health tax rate edges up from 0 to this threshold of 2.1%, the steady state level of capital follows suit. For values of the tax rate larger than this threshold, our model predicts a negative impact of increased public health taxation on capital accumulation. As we can observe in Table 1, an increase of the tax rate larger than 2.1% negatively impacts the capital stocks.

We used the following parameter values for this simulation. A capital-output elasticity $\alpha = 0.4$, in between common estimates for developed and developing countries at $\alpha = 0.33$ and $\alpha = 0.5$.[18] A discount factor $\beta = 0.2$, as in Strulik (2004) and Fanti and Gori (2011a). Pension contributions $\tau_{payg} = 0.15$, as a majority of OECD

[16] A similar multiplier effect of health investment on growth can be found in Chakraborty (2004), Fanti and Gori (2011b) or Fanti and Gori (2014).

[17] Keeping in mind that $\frac{\partial d[h]}{\partial \tau_h} = \left(\frac{w}{1+\tau_h w} + \frac{\tau_h w^2}{(1+\tau_h w)^2}\right)$ and thus increasing in w.

[18] The same argument can be found in Kehoe and Perri (2002) and Fanti and Gori (2011a).

Table 1: Numerical example: the effect of a positive health shock

τ_h	$\bar{\tau}_h$	k^*	Effect on S.S. Level of Capital
0%	0.021	2.4316	
1%	0.021	2.4654	positive
1.5%	0.021	2.4727	positive
2%	0.021	2.4752	positive
2.5%	0.021	2.4738	negative
3%	0.021	2.4693	negative
3.5%	0.021	2.4621	negative
4%	0.021	2.4527	negative

countries have rates between 10% and 20%. An efficiency of health investment at $d_1 = 0.95$, as in Fanti and Gori (2014). An exogenous population growth rate of $n = 0.05$, being the replacement rate in a single-parent model as in Fanti and Gori (2014). In line with Chakraborty (2004), we set $A = 25$. The weight of the standard pension is set at $\rho = 0.25$.[19]

5 Health, disability pensions, and welfare

Let us now look at the welfare effects of health investment in a policy context where pensions are partially health-dependent. More specifically, we're interested in maximizing steady state expected lifetime utility using both tax instruments τ_p and τ_h, but keeping the total tax burden constant.[20] Since in real life a purely PAYG system can to a certain extent always be complemented with a health-dependent dimension -and indeed in many cases already is as argued above- a budget-neutral, second-best exercise of this nature seems justified. Not in the least because raising overall tax levels is far from a feasible policy alternative to many of the debt stricken OECD governments.

Our benevolent government will set an optimal policy pair (τ_h, τ_p) as a first mover, taking into account the decision making of all agents populating the economy. Considering the optimal savings decision in (9), the government therefore knows consumption at a young age will be equal to:

$$c_{1,t} = \frac{(1 + r^e_{t+1})w_t(1 - \tau_h - \tau_p) + (1 + n)w^e_{t+1}[1 - d_{t+1}(1 - \rho)]}{(1 + \beta)(1 + r^e_{t+1})} \tag{14}$$

And similarly, that consumption at an older age will be:

[19]The value of these parameters do not alter the qualitative results of this paper. We choose these values to have results in line with the relevant literature as well as the real world.

[20]Since even an A-Pareto improvement as defined by Golosov (2007) is ruled out because of falling interest rates in the period of reform, we limit ourselves to lifetime utility as a welfare measure.

$$c_{2,t+1} = \frac{\beta w_t(1+r^e_{t+1})(1-\tau_h-\tau_p)-(1+n)w^e_{t+1}[1-d_{t+1}(1-\rho)]}{(1+\beta)} + \frac{(1+\beta)[\rho\tau_p w^e_{t+1}+(1-\rho)(\tau_p(1-d_{t+1})w^e_{t+1})](1+n)}{(1+\beta)} \tag{15}$$

Lastly, public provision in the elderly generation follows:

$$g_{t+1}[d_{t+1}] = (1-\rho)w_{t+1}\tau_p d_{t+1}(1+n) \tag{16}$$

The government can then maximize the following steady-state lifetime indirect utility function $\bar{V}[\tau_p, \tau_h]$, where $\bar{x}[.]$ is the steady state value of the generic variables $x[.]$ defined above:

$$\underset{\tau_h}{Max}\, \bar{V}[\tau_p,\tau_h] = ln\,[\bar{c}_1[\tau_p,\tau_h]] + \beta(ln\,[\bar{c}_2[\tau_p,\tau_h]] + \bar{g}[\tau_p,\tau_h]) \tag{17}$$

Now, to assure budget neutrality, it suffices for the government to optimise with respect to the health tax rate τ_h under the following assumption:

Assumption 1 *Define with $\bar{w}$ and $\bar{w}_{payg}$ the steady-state wage rate under a health-dependent social security system and a purely PAYG pension scheme, respectively. Given parameters, the contribution rate under a health-dependent social security system is such that $\hat{\tau}_p \equiv \tau_p[\tau_h, \tau_{payg}] = \frac{\tau_{payg}\bar{w}_{payg}}{\bar{w}} - \frac{\tau_h}{1+n}$.*

This relation ensures that as health taxation τ_h rises, the contribution rate $\hat{\tau}_p$ decreases proportionally to keep tax revenues constant. To this end, we choose a level of tax revenues accruing to a counterfactual PAYG system as our constant benchmark, so that: $\tau_{payg}\bar{w}_{payg}(1+n) = \hat{\tau}_p\bar{w}(1+n) + \tau_h\bar{w}$ at all times. Total public expenditures $\bar{p}_{payg}$ under this benchmark PAYG pension scheme will then always be identical to those under our the health-dependent social security system, $\bar{p}[\bar{d}] + \bar{g}[\bar{d}] + \bar{h}$.

Now, our optimisation exercise will depend on the general equilibrium effects of marginally increasing health taxation, both on consumption as well as public provision. These are summarised by the following total derivatives:

$$\frac{\partial\bar{c}_1[.]}{\partial\tau_h} = \overbrace{\frac{\partial\bar{c}_1}{\partial\tau_h}}^{-} + \overbrace{\frac{\partial\bar{c}_1}{\partial\hat{\tau}_p}}^{-}\overbrace{\frac{\partial\hat{\tau}_p}{\partial\tau_h}}^{-} + \overbrace{\frac{\partial\bar{c}_1}{\partial\bar{d}}}^{-}\overbrace{\frac{\partial\bar{d}}{\partial\tau_h}}^{+} + \left(\overbrace{\frac{\partial\bar{c}_1}{\partial\bar{w}}}^{+}\overbrace{\frac{\partial\bar{w}}{\partial\bar{k}^*}}^{+} + \overbrace{\frac{\partial\bar{c}_1}{\partial\bar{r}}}^{-}\overbrace{\frac{\partial\bar{r}}{\partial\bar{k}^*}}^{-}\right)\left(\overbrace{\frac{\partial\bar{k}^*}{\partial\tau_h}}^{?} + \overbrace{\frac{\partial\bar{k}^*}{\partial\hat{\tau}_p}}^{-}\overbrace{\frac{\partial\hat{\tau}_p}{\partial\tau_h}}^{-}\right) \tag{18}$$

$$\frac{\partial\bar{c}_2[.]}{\partial\tau_h} = \overbrace{\frac{\partial\bar{c}_2}{\partial\tau_h}}^{-} + \overbrace{\frac{\partial\bar{c}_2}{\partial\hat{\tau}_p}}^{+}\overbrace{\frac{\partial\hat{\tau}_p}{\partial\tau_h}}^{-} + \overbrace{\frac{\partial\bar{c}_2}{\partial\bar{d}}}^{-}\overbrace{\frac{\partial\bar{d}}{\partial\tau_h}}^{+} + \left(\overbrace{\frac{\partial\bar{c}_2}{\partial\bar{w}}}^{+}\overbrace{\frac{\partial\bar{w}}{\partial\bar{k}^*}}^{+} + \overbrace{\frac{\partial\bar{c}_2}{\partial\bar{r}}}^{+}\overbrace{\frac{\partial\bar{r}}{\partial\bar{k}^*}}^{-}\right)\left(\overbrace{\frac{\partial\bar{k}^*}{\partial\tau_h}}^{?} + \overbrace{\frac{\partial\bar{k}^*}{\partial\hat{\tau}_p}}^{-}\overbrace{\frac{\partial\hat{\tau}_p}{\partial\tau_h}}^{-}\right) \tag{19}$$

$$\frac{\partial \bar{g}[.]}{\partial \tau_h} = \overbrace{\frac{\partial \bar{g}}{\partial \bar{d}}}^{+}\overbrace{\frac{\partial \bar{d}}{\partial \tau_h}}^{+} + \overbrace{\frac{\partial \bar{c}_2}{\partial \bar{w}}}^{+}\overbrace{\frac{\partial \bar{w}}{\partial \bar{k}^*}}^{+}\left(\overbrace{\frac{\partial \bar{k}^*}{\partial \tau_h}}^{?} + \overbrace{\frac{\partial \bar{k}^*}{\partial \hat{\tau}_p}}^{-}\overbrace{\frac{\partial \hat{\tau}_p}{\partial \tau_h}}^{-}\right) \tag{20}$$

As equations (18) to (20) clearly demonstrate, the effect of a budget-neutral rise in health investment through increased health taxation is not altogether clear-cut. As pointed out before, much depends on capital accumulation. But even when $\frac{\partial \bar{k}^*}{\partial \tau_h} > 0$ under proposition 2, the outcome still crucially hinges on whether increasing wages outweigh the direct impact of health taxation on consumption, both at a young and an old age. Increasing capital levels also imply a smaller interest rate which may stimulate consumption at the working age, but undercuts it when retired. The drop in pension contribution rates keeping tax revenues constant lastly, brings about the opposite.

Nonetheless, and more analytically put, the government's (second-best) objective can be obtained by maximising:

$$\bar{V} = ln\left[\bar{w}\left(1 - \hat{\tau}_p - \tau_h\right) - \bar{s}\right] + \beta(ln\left[\left(1+\bar{r}\right)\bar{s} + \left(\rho\hat{\tau}_p\bar{w} + (1-\rho)(1-\bar{d})\hat{\tau}_p\bar{w}\right)(1+n)\right] + \bar{g}[\bar{d}]) \tag{21}$$

Which yields the following result for any $0 < \hat{\tau}_p + \tau_h < 1$:

Result 1 *(Second-best health policy) In an economy with a partially health-dependent pension scheme, given assumption 2 and total public expenditures, a value of the health tax τ_h, and therefore of contribution rate $\hat{\tau}_p$, exists that maximises steady-state lifetime indirect utility.*

Result 1 is illustrated in table 2. Using the same parameter values as in section 4, and as observed in the first row of table 2, the health tax rate maximising steady-state indirect utility is given by $\tau_h = 0.055$.[21] This implies a contribution rate for social security of $\hat{\tau}_p = 0.086$. Crucially, driving these results is the exact same multiplier effect as described in section 4. Zooming in on the second row of table 1, rising health tax rates indeed lead to higher capital levels compared to a purely PAYG system, and this keeping total tax burden constant. Wages follow suit in the third row, pushing up consumption in both periods of life in the second panel of the table, even though pensions decrease in the third panel. Also, setting the weight of the universal pension system to a more realistic value of $\rho = 0.75$ doesn't change matters. On the contrary, as table 3 demonstrates, all results carry through with a health tax rate of $\tau_h = 0.081$ maximising lifetime utility.

Now, since we've used a purely PAYG system as a benchmark to keep tax revenues constant in our welfare exercise, tables 2 and 3 also answer a logical follow-up

[21] Since the level of $\bar{\tau}_p$ that allows constant total expenditure in the two social security system depends on health taxation, the maximum admissible heath tax rate that guarantees a positive contribution rate will be given by $\tau_h = 0.139$.

Table 2: Pure vs impure PAYG pension system: $\rho = 0.25$

Variable	*payg*	Health Tax Rate τ_h $\tau_h = 1\%$	$\tau_h = 5\%$	$\tau_h = 5,5\%$	$\tau_h = 10\%$	$\tau_h = 12.5\%$
$\bar{U}$	3.332	3.432	3.559	3.560	3.510	3.456
$\bar{k}^*$	2.432	2.584	2.939	2.968	3.172	3.260
$\bar{w}$	21.402	21.929	23.085	23.178	23.805	24.067
$\bar{c}_1$	15.639	15.995	16.735	16.790	17.150	17.289
$\bar{c}_2$	18.352	18.098	17.530	17.483	17.157	17.015
$\bar{c}_1 + \bar{c}_2$	33.991	34.093	34.265	34.273	34.307	34.304
$\bar{d}$	N/A	0.171	0.509	0.532	0.669	0.713
$\hat{\tau}_p$	15%	13.69%	9.14%	8.61%	3.96%	1.43%
$\bar{h}$	0.000	0.219	1.155	1.275	2.380	3.008
$\bar{p}^u$	3.371	0.788	0.554	0.524	0.248	0.091
$\bar{p}^d$	0.000	1.960	0.816	0.735	0.246	0.078
$\bar{p}^u + \bar{p}^d$	3.371	2.748	1.370	1.259	0.494	0.169
$\bar{g}[\bar{d}]$	0.000	0.404	0.846	0.837	0.497	0.194
Total Exp.	3.371	3.371	3.371	3.371	3.371	3.371

question. Can we improve welfare in the long run by making a standard PAYG pension system partially health-dependent? Result 2 provides the answer.

Result 2 *In an economy with a partially health-dependent pension scheme, and given assumption 1, lifetime steady state welfare levels are higher than in a purely PAYG pension system.*

In other words, even when an optimal combination of health tax and pension contribution rates is politically infeasible -because of e.g. a psychological lower bound on the levels of universal pension benefits- introducing some health-dependent elements still pays off. As the second column of table 3 points out, even setting a health tax of 1% improves lifetime utility considerably. An effect which holds out under higher weights ρ on the universal pension benefit as well.

Table 3: Pure vs impure PAYG pension system: $\rho = 0.75$

Variable	*payg*	Health Tax Rate τ_h $\tau_h = 1\%$	$\tau_h = 5\%$	$\tau_h = 8.1\%$	$\tau_h = 10\%$	$\tau_h = 12.5\%$
$\bar{U}$	3.332	3.370	3.429	3.436	3.434	3.426
$\bar{k}^*$	2.432	2.515	2.777	2.956	3.069	3.218
$\bar{w}$	21.402	21.693	22.571	23.147	23.490	23.940
$\bar{c}_1$	15.638	15.832	16.391	16.741	16.945	17.209
$\bar{c}_2$	18.352	18.206	17.760	17.467	17.294	17.069
$\bar{c}_1 + \bar{c}_2$	33.990	34.038	34.151	34.208	34.239	34.278
$\bar{d}$	N/A	0.170	0.504	0.620	0.666	0.712
$\hat{\tau}_p$	15%	13.85%	9.46%	6.15%	4.14%	1.50%
$\bar{h}$	0.000	0.218	1.129	1.875	2.349	2.992
$\bar{p}^u$	3.371	2.365	1.682	1.122	0.766	0.284
$\bar{p}^d$	0.000	0.655	0.278	0.142	0.085	0.027
$\bar{p}^u + \bar{p}^d$	3.371	3.020	1.960	1.264	0.851	0.311
$\bar{g}[\bar{d}]$	0.000	0.133	0.282	0.232	0.171	0.068
Total Exp.	3.371	3.371	3.371	3.371	3.371	3.371

6 Concluding remarks

Better health not only boosts longevity in itself, it also postpones the initial onset of disability and chronic infirmity to a later age. Taking on the potential impacts of such 'compression of morbidity' on pensions, we introduced a health-dependent dimension to the standard pay-as-you-go (PAYG) pension scheme studied in the literature. Studying the long-term implications of such a system in a simple overlapping generations framework, we've shown under which conditions an increase in public health investment can augment capital accumulation in the long run. Because of this, the combination of health investment with a partially health-dependent PAYG scheme may in fact outperform a purely PAYG system in terms of lifetime welfare, and at identical levels of tax revenue.

Now, in a world where more than two out of five people aged 65 or older report having some sort of functional limitation,[22] these results matter. Indeed, the importance of so called *long-term care* assistance has grown together with the number of dependent elderly.[23] Moreover, because of shifting family patterns -where women enter the labour market rather than caring for older relatives- and a failing private market,[24] this challenge has come to lie with the public sector. Simply extrapolating

[22] These can range from sensory, physical, mental, or self-care disabilities, to difficulties leaving home (see Pestieau and Ponthière (2010)).

[23] This kind of elderly care care can be administered both at home and in various kinds of institutions, including nursing homes and long-stay hospitals (Cremer et al., 2012).

[24] See e.g. Brown and Finkelstein (2007) on the insurance puzzle in the long term care private insurance market, or Pestieau and Sato (2008) on the case for public nursing.

on the basis of existing policies, public expenditures in the EU27 are already expected to increase by 115 per cent on average in the coming 40 years.[25]

In most cases formal long-term care assistance is financed on a PAYG basis, and conditionally assigned using a disability scale identifying various levels of dependency (e.g. the Katz scale). Our emphasis on health-dependent 'disability' pensions then seems justified, and can be seen as extending the standard PAYG pension benefit with a long-term care dimension. Moreover, as this kind of health-dependent benefits are projected to rise in the future, our model lays bare the importance of public health investment. A budget neutral health investment not only improves health itself, but also braces capital accumulation, wages, and consumption through the multiplier effect set in motion by the health-dependent pension scheme. This way, public health investment keeps pensions sustainable in ways a simple contribution increase would otherwise fail to do. Indeed, not only will overall welfare increase given the same tax burden, pension liabilities will gradually erode over time the more we include health status as a factor.

[25]Cremer et al. (2012) base their conjectures on the 2009 'Aging report' of the European Commission, and underline that this projection does not capture the full scale of the policy challenge. Future changes in the number of people receiving informal or no care (which depends on family patterns) are expected to deteriorate, yet assumed constant in their analysis.

Appendix 1: Proof of proposition 1

The proof is done in two steps. First, we prove that the trivial steady state $\bar{k}^* = 0$, the zero equilibrium of the dynamic equation (12), is unstable. Define the right-hand-side of equation (12) as $Z[k]$. Differentiating $Z[k]$ with respect to k gives:

$$Z_k^{'}[k] = \frac{\alpha^2\beta \mathrm{c1} k^{\alpha-1}(1-\tau_h-\tau_p)\left(\mathrm{c1}^2\tau_h^2 k^{2\alpha}(\alpha(\beta+1)+(1-\alpha)\mathrm{c3})+2\mathrm{c1c2}\tau_h k^a+\mathrm{c2}\right)}{(1+n)\left(\mathrm{c2}-\mathrm{c1}\tau_h k^\alpha(\alpha(-\beta+\mathrm{c3}-1)-\mathrm{c3})\right)^2}$$

with $c1$, $c2$ and $c3$ defined in section 3.5 of the main text. Given that $(\mathrm{c1},\mathrm{c2},\mathrm{c3}) > 0$, we observe that $Z_k^{'}[k] > 0$ for any $k > 0$. Since $Z(0) = 0$ and $\lim_{k\to 0^+} Z_k^{'}(k) = +\infty$, it follows that the steady state $\bar{k}^* = 0$ can never be stable.

Second, we prove that there exists an internal solution, $\bar{k}^* > 0$, which is a stable steady state. Rewrite the dynamic equation (12) in steady state, $k = Z[k]$, as:

$$Y_1[k] \equiv k^{1-\alpha} = \frac{\alpha(1+\mathrm{c1}k_t^\alpha\tau_h)(1-\tau_h-\tau_p)\beta\mathrm{c1}}{(1+n)(\mathrm{c2}-\mathrm{c1}k_t^\alpha\tau_h(\alpha(\mathrm{c3}-1-\beta)-\mathrm{c3}))} \equiv Y_2[k]$$

Then observe that $Y_1[0] = 0$, $Y_{1,k}^{'}[k] = (1-\alpha)k^{-\alpha} > 0$ for any $k > 0$, and that $\lim_{k\to+\infty} Y_1[k] = +\infty$. Define $Y_2[0] = \frac{\alpha\mathrm{c1}(1-\tau_h-\tau_p)\beta}{(1+n)\mathrm{c2}}$, :

$$\lim_{k\to+\infty} Y_2[k] = \frac{\alpha\mathrm{c1}(1-\tau_h-\tau_p)\beta}{(1+n)(\mathrm{c3}(1-\alpha)+\alpha(1+\beta))}$$

and

$$Y_{2,k}^{'}[k] = \frac{\alpha^2\beta\mathrm{c1}^2\tau_h k^{\alpha-1}(\tau_h+\tau_p-1)(-\mathrm{c2}+\alpha(\beta-\mathrm{c3}+1)+\mathrm{c3})}{(1+n)\left(\mathrm{c2}-\mathrm{c1}\tau_h k^\alpha(\alpha(-\beta+\mathrm{c3}-1)-\mathrm{c3})\right)^2}$$

Using $\mathrm{c2} = \alpha(1+\beta)+\tau_p(1-\alpha)$ and $\mathrm{c3} = \tau_p(1-d_1(1-\rho))$, we observe that the denominator of $Y_{2,k}^{'}[k]$ is always positive. The numerator can be written as: $(1-\alpha)\alpha^2 c1^2 d_1 k^{\alpha-1}\tau_h\tau_p(1-\tau_h-\tau_p)\beta(1-\rho)$. This expression is positive for any $k > 0$, implying that $Y_{2,k}^{'}[k] > 0$. Moreover, notice that $Y_2(0) < \lim_{k\to+\infty} Y_2[k]$ when $(1-\alpha)d_1\tau_p(1-\rho) > 0$. Given restrictions on parameters, the latter condition is always verified. It follows that for any $k > 0$, $Y_1[k] = Y_2[k]$ only once at $\bar{k}^* > 0$, characterising the asymptotically stable steady state. ■

Appendix 2: Proof of proposition 2

Define the relation between steady state of capital $\bar{k}$ and health taxation τ_h as follows:[26] $G[\bar{k}, \tau_k] = \bar{k} - Z[\bar{k}, \tau_h]$ with $Z[\bar{k}, \tau_h]$ defined as the right-hand-side of equation (12) in steady state. We apply the implicit function theorem to derive the effect of health taxation, τ_h, on capital $\bar{k}$:

$$\bar{k}'_{\tau_h}[\tau_h] = -\frac{\frac{\partial G[\bar{k},\tau_h]}{\partial \tau_h}}{\frac{\partial G[\bar{k},\tau_h]}{\partial \bar{k}}} = -\frac{A}{B} \tag{22}$$

Where A in expression(22) denotes:

$$A = \alpha\beta \mathrm{c1}\bar{k}^{\alpha}\left(\mathrm{c2}\left(\mathrm{c1}\bar{k}^{\alpha}(2\tau_h + \tau_p - 1) + 1\right) - \mathrm{c1}\bar{k}^{\alpha}(\alpha(-\beta + \mathrm{c3} - 1) - \mathrm{c3})\left(\mathrm{c1}\tau_h^2\bar{k}^{\alpha} - \tau_p + 1\right)\right) \tag{23}$$

With c1, c2 and c3 defined in section 3.5 of the main text. Similarly, B is equal to:

$$B = \alpha^2\beta \mathrm{c1}\bar{k}^{\alpha-1}(\tau_h + \tau_p - 1)\left(\mathrm{c1}^2\tau_h^2\bar{k}^{2\alpha}(\alpha(\beta - \mathrm{c3} + 1) + \mathrm{c3}) + 2\mathrm{c1c2}\tau_h\bar{k}^{\alpha} + \mathrm{c2}\right) +$$
$$+ (1+n)\left(\mathrm{c2} - \mathrm{c1}\tau_h\bar{k}^{\alpha}(\alpha(-\beta + \mathrm{c3} - 1) - \mathrm{c3})\right)^2 \tag{24}$$

The derivative $\bar{k}'_{\tau_h}[\tau_h]$ is equal to zero when the numerator is equal to zero. Solving in terms of τ_h, allows us to observe that the numerator is zero if $\tau_h = \bar{\tau}_h$, with:

$$\bar{\tau}_h = \frac{\bar{k}^{-2\alpha}\left(\mathrm{c1c2}\bar{k}^{\alpha} \pm \sqrt{\mathrm{c1}^2\bar{k}^{2\alpha}(\mathrm{c2} + (\mathrm{c1}(\tau_p - 1)\bar{k}^{\alpha})(\alpha(-\beta + \mathrm{c3} - 1) - \mathrm{c3})\left(\alpha(-\beta + \mathrm{c3} - 1) - \mathrm{c3}\right) + \mathrm{c2})}\right)}{\mathrm{c1}^2(\alpha(-\beta + \mathrm{c3} - 1) - \mathrm{c3})}$$

Note that the denominator of the threshold $\bar{\tau}_h$ is always negative, so that $\bar{\tau}_h$ can be positive only when the numerator is negative. Since the term below the square root is positive and imaginary solutions are therefore ruled out, a positive threshold $\bar{\tau}_h$ can be obtained by keeping the minus sign before the square root. In this case, the threshold will be positive when: $\bar{k} > \tilde{k} \equiv \left(\frac{\tau_p + \alpha(1 - \tau_p + \beta)}{\alpha(\alpha-1)^2 d_1(1-\tau_p)\tau_p(1-\rho)}\right)^{\frac{1}{\alpha}}$. Moreover, $\bar{\tau}_h$ is is also smaller than 1. To prove this statement, it is sufficient to observe that $\bar{\tau}_h < 1$ when $\beta > \tilde{\beta}$. The latter condition is always verified since $\beta > 0$ by assumption and $\tilde{\beta} \equiv \frac{\bar{k}^{-\alpha}\left(\mathrm{c1}(\alpha(\mathrm{c3}-1)-\mathrm{c3})\bar{k}^{\alpha}\left(\mathrm{c1}\bar{k}^{\alpha} - \tau_p + 1\right) - \mathrm{c2}\left(\mathrm{c1}(\tau_p+1)\bar{k}^{\alpha} + 1\right)\right)}{\alpha \mathrm{c1}\left(\mathrm{c1}\bar{k}^{\alpha} - \tau_p + 1\right)} < 0$.

In order to prove that $\frac{\partial \bar{k}}{\partial \tau_h} > 0$ when $\tau_h < \bar{\tau}_h$, we have to consider the sign of the numerator and denominator of $\bar{k}'_{\tau_h}[\tau_h]$, as expressed by (22). First, and after some manipulation, the denominator in (22) is always positive for any $\bar{k} > 0$. Solving the denominator in terms of $\bar{k}$ we derive $\tilde{\tilde{k}} \equiv \left(-\frac{c2}{c1\tau_h(c3(1-\alpha)+\alpha(1+\beta))}\right)^{\frac{1}{\alpha}} < 0$. Notice that the equation of the denominator crosses the x-axis once at $\tilde{\tilde{k}}$. Considering that at $\bar{k} = 0$ the denominator of $\bar{k}'_{\tau_h}[\tau_h]$ reduces to $(1+n)\mathrm{c2}^2$, the equation is necessarily increasing. It follows that the denominator is strictly increasing in $\bar{k}$ and is always positive under assumption 1. Thus, the sign of $\bar{k}'_{\tau_h}[\tau_h]$ will depend on the sign of the numerator in (22).

[26] For simplicity, $*$ is omitted.

Define for simplicity $I[\bar{k}] = \mathrm{c}1\bar{k}^{\alpha}$ and $X = -\mathrm{c}3 + \alpha(\mathrm{c}3 - 1 - \beta)$. Then, observe that the numerator of $\bar{k}^{'}_{\tau_h}[\tau_h]$ expressed by (22) is zero when the health tax rate $\tau_h = \frac{\mathrm{c}2 I[\bar{k}] \pm \sqrt{I[\bar{k}]^2(\mathrm{c}2+X)(\mathrm{c}2+(\tau_p-1)XI[\bar{k}])}}{XI[\bar{k}]^2}$, that is when $\tau_h = \bar{\tau}_h$ as defined above. The fact that two solutions exist, indicates that the numerator of equation $\bar{k}^{'}_{\tau_h}[\tau_h]$ is a parabola. Rewriting the numerator of (22) as follows:

$$\alpha I[\bar{k}](\mathrm{c}2 + I[\bar{k}]\mathrm{c}2(2\tau_h + \tau_p - 1) - I[\bar{k}](1 + I[\bar{k}]\tau_h^2 - \tau_p)X)\beta \tag{25}$$

and deriving with respect to τ_h, we get:

$$2\alpha I[\bar{k}]^2(c_2 - \tau_h X I[\bar{k}])\beta$$

Since $X < 0$, the derivative is positive when $\tau_h > 0$ and the critical point, $\frac{\mathrm{c}2}{XI[\bar{k}}$, negative. Finally, observe that the second derivative, $-2\alpha X I[\bar{k}]^3\beta$ is always positive. It follows that the critical point of equation (25) is a minimum and the branch of the parabola in the domain $\tau_h > 0$ always increasing, crossing the x-axis when $\tau_h = \bar{\tau}_h$.

As proved above only one of the two solutions of $\bar{k}^{'}_{\tau_h}[\tau_h] = 0$ can be positive (if $\bar{k} > \tilde{k}$) and smaller than one. Then, in the domain $\tau_h > 0$, the sign of the numerator in (22) will be negative for any $\tau_h < \bar{\tau}_h$ and positive otherwise. Since the sign of the re-worked denominator in (22) was always positive, we observe that $\bar{k}^{'}_{\tau_h}[\tau_h] = -\frac{\leq 0}{>0}$ i.e. > 0 if $\tau_h < \bar{\tau}_h$ and that $\bar{k}^{'}_{\tau_h}[\tau_h] = -\frac{\geq 0}{>0}$ i.e. < 0 if $\tau_h > \bar{\tau}_h$. ■

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